ABSTRACT: This paper shows how an additional provision of reactive power influences the economically optimized sizing of inverters for photovoltaic systems in Germany. At first, the results of an encompassing survey among 934 photovoltaic system operators show the current inverter sizing experiences for a region in South-East Bavaria. In a second step an analytical approach for the determination of the economically optimized inverter sizing is introduced, considering active as well as reactive power provision. The simulation results, shown in this paper, are based on real DC measurement data of modules at a test site in Germany, with a time resolution of 1 minute for a period of four years. It is shown that an additional reactive power provision leads to higher economically optimized inverter sizing compared to pure active power feed-in, depending on the actual reactive power supply method and the minimum power factor required.

Keywords: Photovoltaic Inverter, Reactive Power, Inverter Sizing

1 INTRODUCTION

Today, the installed photovoltaic (PV) capacity in Germany counts more than 17 GW (status April 2011), of which 80% is connected to low voltage systems [1]. From August 2011 on, the provision of reactive power can be required from all newly installed PV systems, which are connected to low voltage levels [2].

The technical effectiveness of providing reactive power to reduce critical voltage rises has been analyzed in recent publications [3], [4]. Also the economic benefits of reactive power provision by PV systems for the distribution system operation were analyzed [5], [6].

Besides the positive effects of reactive power on limiting local voltage rises, additional reactive power currents lead to higher loadings of PV inverters and hence might affect the economically optimized inverter sizing of future PV systems. An approach to PV inverter sizing under consideration of an additional reactive power provision has already been analyzed in [7]. Further improvements of this approach are presented in this paper.

In Section 2, the results of a survey among 934 PV plant operators will highlight the experience with current PV inverter sizing practices in South-East Bavaria. From Section 3 on, this paper focuses on the economically optimized inverter sizing under the consideration of an additional reactive power provision, based on 1 minute DC power measurement data for a PV test site in Kassel (compare Figure 1). The simulation assumptions for the calculation of the economically optimized inverter sizing will be introduced first, followed by an overview of the different cost categories for an economically optimized inverter sizing. Finally, the optimization results for PV systems in Kassel, under current regulatory framework conditions are shown.

Figure 1: Installed PV-Capacity per ZIP-Code Area in Germany with markers for the investigated locations (Figure according to Saint-Drenan, Fraunhofer IWES).

2 INVERTER SIZING IN THE PAST – FIELD EXPERIENCE WITH GERMAN PV-INSTALLATIONS

The optimum sizing of the inverter depends on site and system-specific parameters. The meteorological ratings, irradiation and ambient temperature can be seen as site-specific, whereas the tilt angle, the orientation and the ventilation are system-specific. The system designer has to choose the optimal sizing for each system considering all the parameters. Optimally tilted, south-oriented, free-standing and therefore well ventilated PV installations in southern Germany will need a higher sizing factor.
A study within the “Netz der Zukunft” (“The Grid of the Future”) project of E.ON Bayern AG, the Technische Universität München and the University of Applied Sciences Munich, shows the variability and the final inverter sizing of PV systems in the south-eastern part of Germany [8], [9].

For the study, 934 PV systems in four different counties in South-East Bavaria were analyzed. The sizing ratios of these inverters are defined by the rated power of the generators (P_{PV_STC}) and the continuous output power rating of the inverters (PWR_AC_NOM). The sizing ratio limits the maximum output of the entire PV system. The maxima of the frequency distribution of the sizing ratios, which are shown in Figure 2, are between 0.85 and 0.95 for all kinds of the investigated system sizes. The systems with a continuous output power rating of above 100 kW are shown with a dashed line, since there are only 12 systems in this class.

![Figure 2: Frequency distribution of current inverter sizing in South-East Bavaria.](image)

It can be seen in Table 1 that the sizing ratio for smaller systems is below the overall average and shows a rising trend towards taller PV systems.

<table>
<thead>
<tr>
<th>System Size</th>
<th>Sizing Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up to 5 kW</td>
<td>0.88</td>
</tr>
<tr>
<td>5 kW to 10 kW</td>
<td>0.88</td>
</tr>
<tr>
<td>10 kW to 30 kW</td>
<td>0.90</td>
</tr>
<tr>
<td>30 kW to 100 kW</td>
<td>0.90</td>
</tr>
<tr>
<td>100 kW to 400 kW</td>
<td>0.93</td>
</tr>
<tr>
<td>Total</td>
<td>0.89</td>
</tr>
</tbody>
</table>

The survey shows that PV inverters in the region of South-East Bavaria are often sized smaller than the respective rated DC power of the generators. However, it has to be taken into account that the inverters in the survey were sized for the feed-in of active power only.

In the following section, an approach for determining the economically optimized inverter sizing under the consideration of an additional reactive power provision is introduced.

### 3.1 SIMULATION ASSUMPTIONS

In order to determine the influence of an additional reactive power provision on the economically optimized sizing of PV inverters, an inverter model was set up in Matlab®. The inverter model reads the DC power data from the modules and calculates the active and reactive power output of the inverter, taking the respective reactive power supply method and the inverter efficiency into account.

**DC power time series**

For this investigation, measured DC power output data for the years 2007 - 2010 is available for Kassel (51.19° lat., 9.29° long.), Germany. The measurement data has a time resolution of 15 second, which is averaged to 1 minute data to improve the speed of the simulation. Figure 3 shows the DC energy yield of the test site for 2008, scaled for a 1 kWp monocrystalline silicon module.

![Figure 3: Energy yield of the used DC power measurement data with a timely resolution of 1 minute.](image)

The inverter model

The parameterization of the inverter model is based on the specifications of the Sunny Minicentral TLRP 11000 from SMA [10]. Figure 4 shows the inverter loss coefficients as derived from the product sheet and the fitted second degree polynomial function according to Equation 1 [11]. With this function, the inverter losses can be determined depending on the inverter active power output.

\[
\text{Losses} = c_{\text{self}} + c_v \cdot \text{P}_{\text{AC}} + c_f \cdot \text{P}_{\text{AC}}^2 
\]

(1)

Here, \( c_{\text{self}} \) stands for the self-consumption of the inverter, \( c_v \) for the voltage dependant losses and \( c_f \) for the current dependant losses.

For the calculation of the economically optimized inverter sizing under consideration of reactive power, the reactive currents also have to be taken into account for the determination of the inverter losses. According to [12] Equation 1 can be also written as a polynomial function depending on the apparent power \( S \), if the active power \( P \) and reactive power output \( Q \) are given.

\[
S = \sqrt{P^2 + Q^2} 
\]

(2)

\[
\text{Losses} = c_{\text{self}} + c_v \cdot S + c_f \cdot S^2 
\]

(3)
3.2 ECONOMICALLY OPTIMIZED INVERTER SIZING – SIMULATION RESULTS

This section shows the simulation results for the economically optimized inverter sizing under consideration of reactive power, based on the introduced assumptions. The simulations are conducted for the period of one year and also for the average of the four year data sample.

In principal, the optimization process is to weight the investment costs for the inverter against opportunity costs caused by a reduced power feed-in due to the inverter sizing. On the one hand a generously sized inverter might guarantee the feed-in of the maximum energy yield, but on the other hand would lead to high investment costs for the inverter hardware. While the investment costs for the PV inverter are predetermined by market prizes, the opportunity costs, due to a reduced power feed-in, depend on several influencing factors during the lifetime of the PV plant (feed-in tariff, location, alignment of PV panels, weather conditions, etc.). Studies on the optimized inverter sizing under different framework conditions have already been carried out in [13] and [14].

A new aspect, which also influences the PV power feed-in and hence the opportunity costs is an additional reactive power provision. By providing a certain amount of reactive power, the capacity of the inverter for feeding-in active power will be limited further. The additional opportunity losses \( \Delta P_{\text{op},Q\neq 0} \) caused by reactive power can be calculated by Equation (4), considering the maximum inverter capacity \( S_{\text{max}} \), the reactive power provision \( Q(t) \) and the rated additional inverter losses \( \Delta p_L(t) \) [kW/kVA], due to the reactive power provision.

\[
\Delta P_{\text{op},Q\neq 0}(t) = S_{\text{max}} \left( 1 - \frac{Q(t)}{S_{\text{max}}} \right) - \sqrt{S_{\text{max}}^2 - Q(t)^2} \quad \text{[kW]} \tag{4}
\]

A detailed mathematical description of the single cost categories can be found in [7].

Figure 6 shows the characteristics of the different cost categories (additional costs compared to an inverter sizing of 1 kVA/kWp and pure active power feed-in), depending on the sizing of the PV inverter for the location Kassel (year 2008). Here, the inverter sizing is related to the maximum DC power generation of the installed modules. In this example, the additional total costs and the point of the cost-optimal inverter sizing are depicted for the operation under consideration of a fixed power factor \( \cos \phi = 0.9 \). It can be seen that the economically optimized inverter sizing for this particular example is 0.91 kVA/kWp, assuming inverter costs of 350 €/kVA and a feed-in tariff of 28.74 c€/kWh. Furthermore, the characteristics of the opportunity costs with and without the provision of reactive power are depicted. The deviation shows the influence of the additional reactive power provision on the opportunity costs.

### Table 2: Overview about the simulation assumptions.

<table>
<thead>
<tr>
<th>Module tilt angle</th>
<th>30°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module orientation</td>
<td>0° (south)</td>
</tr>
<tr>
<td>Module technology</td>
<td>Monocrystalline silicon</td>
</tr>
<tr>
<td>PV inverter type</td>
<td>SMA Sunny Minicentral TLRP 11000 with reactive power control</td>
</tr>
<tr>
<td>Max. inverter efficiency / Euro-eta</td>
<td>97.7% / 97.2%</td>
</tr>
<tr>
<td>Reimbursement for PV energy</td>
<td>0.2874 €/kWh</td>
</tr>
<tr>
<td>Inverter costs</td>
<td>300 €/kVA – 500 €/kVA</td>
</tr>
<tr>
<td>Reactive power supply method</td>
<td>Fixed cosφ 0.9 (if not stated otherwise)</td>
</tr>
<tr>
<td>Inverter life time</td>
<td>10 years</td>
</tr>
<tr>
<td>Discount rate</td>
<td>5%</td>
</tr>
</tbody>
</table>

![Figure 4: Inverter losses characteristic and the derived polynomial function.](image1)

![Figure 5: AC power feed-in as a result of the inverter model for a summer day in Kassel in 2008.](image2)

![Figure 6: Economically optimized inverter sizing under consideration of a fixed power factor of 0.9 for the recorded data of Kassel (year 2008).](image3)
power output of the PV generator. A possible characteristic which is used for the simulations is shown in Figure 7. There are also other methods like a Q(U)-droop function or a fixed reactive power output, though they are not considered in this investigation.

Figure 7: Possible reactive power provision characteristic, depending on the power output of the generator.

PV plants connected to the medium voltage level in Germany have to be capable of providing a minimum power factor of 0.95 leading and lagging [15]. For PV plants connected to the low voltage level, the minimum power factor varies between 0.95 and 0.9 leading and lagging, depending on the installed PV capacity [2].

Figure 8 gives an overview of the economically optimized inverter sizing under the consideration of different reactive power supply methods. The specific investment costs for the inverter are set to 350 €/kVA. The distinguished operation modes are:

- Only active power provision
- Fixed power factor of 0.9
- Fixed power factor of 0.95
- Cosφ(P) characteristic with a minimum power factor of 0.9
- Cosφ(P) characteristic with a minimum power factor of 0.95

Comparing the inverter sizing for the different efficiencies leads to the following conclusion. The higher the efficiency of the inverter leads to higher optimal inverter sizing for given specific inverter costs. Here, the trade-off between additional investment costs for a higher sized inverter with higher efficiency and the higher lost opportunity costs for the inverter with the lower efficiency becomes visible. Lower efficient inverters with lower investment costs in general and lower investment costs for optimal sizing lead to cost savings on the one hand. On the other hand the lost opportunity cost increase with a lower efficiency. This trade-off will be addressed in future research.

Figure 10 shows the influence of the feed-in tariffs on the economically optimized inverter sizing. The German feed-in tariffs over the last years as well as a projection into the future have been analysed here. As the value of the PV energy gets lower (feed-in tariff degression), higher feed-in losses can be taken into account in order to save additional investment costs by reducing the sizing of the inverter. For this example, a fixed power factor of 0.9 and specific inverter costs of 350 €/kVA are assumed.

The comparison of the different efficiencies of the inverter shows again that the higher efficiency leads to higher inverter sizing since this enables more energy to
get reimbursed. Additionally, the analysis shows that reactive power provision requires a higher optimal inverter sizing than pure active power supply like Figure 8 pointed out.

Comparing the economically optimized inverter sizing for the location Kassel and the results of the field test in South-East Bavaria shows a difference between these two areas (considering only pure active power feed-in). This might be due to different irradiation conditions in both regions. Additionally, the influence of the feed-in tariff influences the sizing as Figure 10 points out. The inverters in the survey were installed in the past, where higher feed-in tariff were still in place. This decreases the difference between the simulated sizing and the survey sizing. However, to wholly determine the influence of different locations on the economically optimized inverter sizing, additional validated measurement data from additional locations should be used. Therefore, additional calculations will be done in future.

The results of the simulations in Section 3 are built upon DC power measurement data of a PV site in Kassel, Germany. In cases where, instead of the measured DC power, the measured solar irradiation and the ambient temperature (preferably the module temperature) are available, a detailed model for the behavior of the PV module is also required. An approach for such a model is described in the following section.

4 Detailed model for the calculation of the PV module output power

A detailed model for the calculation of the PV module output power is necessary where instead of the measured DC power output, the solar irradiation and the ambient temperature are available.

The proposed module model in this investigation uses the solar irradiation \( G \) and module temperature \( T \) time series to calculate the module DC power output according to a modification of the model presented by [16]. The original model was developed for one hour irradiation values, which were used to calculate the module efficiency \( \eta \):

\[
\eta_{\text{mp}}(G,T) = (a_1 + a_2 G + a_3 G^2) \left(1 + a(T-25^\circ\text{C})\right) \tag{5}
\]

with \( \eta_{\text{mp}} = \frac{P_{\text{mp}}}{G \cdot A_{\text{module}}} \) \( \text{(6)} \)

The temperature coefficient \( a \) is provided by the module producer. The parameters \( a_1 \), \( a_2 \) and \( a_3 \) are derived by performing a parameter fit with measurement data. This high level approach entails some weaknesses when analyzing high resolution time data. The DC power output gets underestimated for high solar radiation and overestimated for lower strength of radiation. The authors extended and modified this approach to enable an accurate parameter fit for high resolution measurement data, e.g. 1 sec. to 1 min. time data. Depending on the irradiation \( G \), the parameter fit is performed for \( i \) irradiation classes. We derive \( i \) efficiencies \( \eta_i \) depending on the irradiation \( G \), e.g. \( G > 800 \text{ W/m}^2 \).

Figure 11 shows examples of module efficiency curves using the original parameter fit method and the sectional parameter fit method. Compared to the measured radiation data, it becomes visible that the sectional fit improves the accuracy of the model. The example is based on 15 seconds measurement data for the same location as described in Table 2. The irradiation data was measured using the ISET-sensor. The temperature data is the measured module temperature.

The measurement data was derived using PV module with an efficiency of \( \eta_{\text{STC}} = 13.7\% \) stated by the manufacturer. In the simulation we derive an efficiency of 12.9% for the sectional parameter fit method and 12.7% for one parameter fit method. The difference to the manufacturer efficiency can be explained through the aging of the module. The data used for the simulation was measured between 2007-2010; the module had been installed in 2003. Additionally, this approach allows determining the actual power of the module for each year in order to perform a proper scaling to 1 kWp for the analysis in section 3.

This approach has been validated by comparing the simulated DC power output of the module model with the actual measured DC power output of the real module at Kassel. For the annual energy yield the difference between the model and the measurement data for the four described years is in average lower than 0.80%.

Comparing the economically optimized inverter sizing using the measured DC power data and the introduced module model as well as the original one-fit model leads to the following results. The results are based on the standard test case: a fixed reactive power factor of 0.9, inverter investment cost of 350 €/kVA and a feed-in tariff of 28.74 €/kWh.

For all years the optimal inverter sizing received from the new module model was the same as the optimal inverter sizing for measured data. For pure active power supply, the difference for the total costs/savings is less
Comparing the one-fit model to the measured data shows that the one-fit model leads to minimal higher inverter sizing in two cases for the years 2007-2010 for reactive power and real power supply. The difference is negligible small: in year 2007 the measurement data leads to a sizing of 0.89 while the one-fit model leads to a sizing of 0.90. The total costs/savings difference show a higher difference than the sectional-fit model did. Here, the offset ranges from 0.10-0.45 €/(kVA yr). Nevertheless, the difference is small.

The sectional-fit method improves the PV module modelling only slightly in this case, further examples with higher benefits are under investigation right now and will be presented in future research.

5 CONCLUSIONS
A survey among 934 PV plant operators in South-East Bavaria shows that the average inverter sizing in this region is 0.89 kVA/kWp, with a slightly increasing tendency towards PV systems with more than 100 kWp of rated module power. An analytical determination of the economically optimized inverter sizing, using the recorded DC measurement data of a PV test site in Kassel, leads to lower optimized inverter sizing. The difference between the average of the survey and the simulation might be explained by the different irradiation conditions in both areas as well as the influence of the inverter efficiency, value of feed-in tariffs and inverter investment costs. However, before comparing the current sizing practices of a certain region in Germany with its theoretical optimum, additional simulations with validated measurement data from the particular region have to be conducted.

The optimal sizing of PV inverters has to be increased, if an additional reactive power provision is required. Depending on the reactive power provision method and the minimum power factor, the additional sizing varies for the investigated location (Kassel, year 2008, specific inverter costs of 350 €/kVA) between +3.7% for a cos(P) method with a minimum power factor of 0.9 and +11.0% for a fixed power factor of 0.9.

Moreover, an improved approach for the calculation of the PV module DC power output, based on the solar irradiation and the ambient temperature was introduced that improves the precision at different irradiation sections significantly.

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